Основные тригонометрические тождества

$\sin^2\alpha + \cos^2\alpha = 1$

$\mathrm{tg}\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

$\mathrm{ctg}\alpha = \dfrac{\cos\alpha}{\sin\alpha}$

$\mathrm{tg}\alpha\,\mathrm{ctg}\alpha = 1$

$1 + \mathrm{tg}^2\alpha = \dfrac{1}{\cos^2\alpha}$

$1 + \mathrm{ctg}^2\alpha = \dfrac{1}{\sin^2\alpha}$

Формулы суммы и разности

$\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

$\sin\alpha - \sin\beta = 2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

$\cos\alpha - \cos\beta = -2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

$\mathrm{tg}\alpha + \mathrm{tg}\beta = \dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$

$\mathrm{tg}\alpha - \mathrm{tg}\beta = \dfrac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}$

$\mathrm{ctg}\alpha + \mathrm{ctg}\beta = \dfrac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$

$\mathrm{ctg}\alpha - \mathrm{ctg}\beta = \dfrac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$

Формулы двойного аргумента

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\cos 2\alpha = 1 - 2\sin^2\alpha$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$\mathrm{tg}\,2\alpha = \dfrac{2\,\mathrm{tg}\alpha}{1-\mathrm{tg}^2\alpha}$

$\mathrm{ctg}\,2\alpha = \dfrac{\mathrm{ctg}^2\alpha - 1}{2\,\mathrm{ctg}\alpha}$

$\sin 3\alpha = 3\sin\alpha\cos^2\alpha - \sin^3\alpha$

$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

$\cos 3\alpha = \cos^3\alpha - 3\sin^2\alpha\cos\alpha$

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

Формулы произведений

$\sin\alpha\sin\beta = -\dfrac{1}{2}(\cos(\alpha+\beta) - \cos(\alpha-\beta))$

$\cos\alpha\cos\beta = \dfrac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$

$\sin\alpha\cos\beta = \dfrac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$

$\cos\alpha\sin\beta = \dfrac{1}{2}(\sin(\alpha+\beta) - \sin(\alpha-\beta))$

Формулы сложения

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\mathrm{tg}(\alpha+\beta) = \dfrac{\mathrm{tg}\alpha + \mathrm{tg}\beta}{1 - \mathrm{tg}\alpha\,\mathrm{tg}\beta}$

$\mathrm{tg}(\alpha-\beta) = \dfrac{\mathrm{tg}\alpha - \mathrm{tg}\beta}{1 + \mathrm{tg}\alpha\,\mathrm{tg}\beta}$

$\mathrm{ctg}(\alpha+\beta) = \dfrac{\mathrm{ctg}\alpha\,\mathrm{ctg}\beta - 1}{\mathrm{ctg}\alpha + \mathrm{ctg}\beta}$

$\mathrm{ctg}(\alpha-\beta) = \dfrac{\mathrm{ctg}\alpha\,\mathrm{ctg}\beta + 1}{\mathrm{ctg}\alpha - \mathrm{ctg}\beta}$

Формулы половинного аргумента

$\sin^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{2}$

$\cos^2\dfrac{\alpha}{2} = \dfrac{1+\cos\alpha}{2}$

$\mathrm{tg}^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{1+\cos\alpha}$

$\mathrm{ctg}^2\dfrac{\alpha}{2} = \dfrac{1+\cos\alpha}{1-\cos\alpha}$

$\sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$

$\cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$

$\mathrm{tg}\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}} = \dfrac{\sin\alpha}{1+\cos\alpha}$

$\mathrm{ctg}\dfrac{\alpha}{2} = \dfrac{1+\cos\alpha}{\sin\alpha}$

	$-\alpha$	$90° - \alpha$	$90° + \alpha$	$180° - \alpha$	$180° + \alpha$	$270° - \alpha$	$270° + \alpha$	$360° - \alpha$	$360° + \alpha$
	$-\alpha$	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi-\alpha$	$2\pi+\alpha$
sin	-sin α	cos α	cos α	sin α	–sin α	–cos α	–cos α	-sin α	sin α
cos	cos α	sin α	–sin α	–cos α	–cos α	–sin α	sin α	cos α	cos α
tg	-tg α	ctg α	–ctg α	–tg α	tg α	ctg α	–ctg α	–tg α	tg α
ctg	-ctg α	tg α	–tg α	–ctg α	ctg α	tg α	-tg α	-ctg α	ctg α

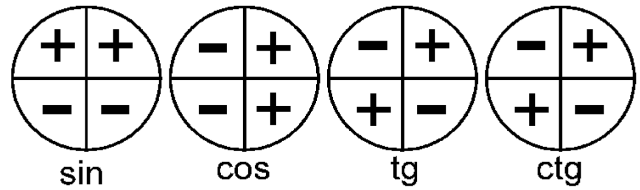
Формулы понижения степени

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\cos^3\alpha = \frac{3\cos\alpha + \cos 3\alpha}{4}$$

$$\sin^3\alpha = \frac{3\sin\alpha - \sin 3\alpha}{4}$$



Решения тригонометрических уравнений

$$\cos x = a; x = \pm\arccos a + 2\Pi n; n \in Z$$

$$\cos x = 0; x = \frac{\Pi}{2} + \Pi n; n \in Z$$

$$\cos x = 1; x = 2\Pi n; n \in Z$$

$$\cos x = -1; x = \Pi + 2\Pi n; n \in Z$$

$$\sin x = a; x = (-1)^n \arcsin a + \Pi n; n \in Z$$

$$\sin x = 0; x = \Pi n; n \in Z$$

$$\sin x = 1; x = \frac{\Pi}{2} + \Pi n; n \in Z$$

$$\sin x = -1; x = -\frac{\Pi}{2} + \Pi n; n \in Z$$

$$\mathrm{tg}x = a; x = \mathrm{arctg}a + \Pi n; n \in Z$$

$$\mathrm{ctg}x = a; x = \mathrm{arcctg}a + \Pi n; n \in Z$$

Выражение формул через тангенс

$$\sin\alpha = \frac{2\,\mathrm{tg}\frac{\alpha}{2}}{1+\mathrm{tg}^2\frac{\alpha}{2}} \qquad \cos\alpha = \frac{1-\mathrm{tg}^2\frac{\alpha}{2}}{1+\mathrm{tg}^2\frac{\alpha}{2}}$$

$$\mathrm{tg}\alpha = \frac{2\,\mathrm{tg}\frac{\alpha}{2}}{1-\mathrm{tg}^2\frac{\alpha}{2}}$$

Формулы сокращенного умножения

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

α	0	30	45	60	90	180	270	360
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$tg\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	–	0	–	0
$ctg\alpha$	–	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	–	0	–

$$ax^2 \pm bx \pm c = 0$$

$$D = b^2 - 4ac \qquad \frac{D}{4} = \left(\frac{b}{2}\right)^2 - ac$$

$$x_{1;2} = \frac{-b \pm \sqrt{D}}{2a} \qquad x_{1;2} = \frac{-b/2 \pm \sqrt{D/4}}{a}$$

При D=0: $x = -\frac{b}{2a}$

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$